P-III/(1+1+1)/H/20(N)

2020

MATHEMATICS (Honours)

Paper Code : VI-A & B

(New Syllabus)

Full Marks : 100

Time : Four Hours

Important Instructions for Multiple Choice Question (MCQ)

- Write Subject Name and Code, Registration number, Session and Roll number in the space provided on the Answer Script.

  Example : Such as for Paper III-A (MCQ) and III-B (Descriptive).

  Subject Code : | III | A | & | B |

  Subject Name : [ ]

- Candidates are required to attempt all questions (MCQ). Below each question, four alternatives are given [i.e. (A), (B), (C), (D)]. Only one of these alternatives is 'CORRECT' answer. The candidate has to write the Correct Alternative [i.e. (A)/(B)/(C)/(D)] against each Question No. in the Answer Script.

  Example — If alternative A of 1 is correct, then write :
  1. — A

- There is no negative marking for wrong answer.

মাল্টিপল চয়েস প্রশ্নের (MCQ) জন্য জরুরী নির্দেশাবলী

- উত্তরপত্রে নির্দেশিত স্থানে বিষয়ের (Subject) নাম এবং কোড, রেজিস্ট্রেশন নম্বর, সেশন এবং রোল নম্বর লিখতে হবে।

উদাহরণ — যেমন Paper III-A (MCQ) এবং III-B (Descriptive)।

Subject Code : | III | A | & | B |

Subject Name : [ ]

- পরীক্ষার্থীদের সবগুলি প্রশ্নের (MCQ) উত্তর দিতে হবে। প্রতিটি প্রশ্নে চারটি করে সম্ভাব্য উত্তর, যথাক্রমে (A), (B), (C) এবং (D) করে দেওয়া আছে। পরীক্ষার্থীকে তার উত্তরের স্বপক্ষে (A) / (B) / (C) / (D) সঠিক বিকল্পটিকে প্রশ্ন নম্বর উল্লেখসহ উত্তরপত্রে লিখতে হবে।

উদাহরণ — যদি **1** নম্বর প্রশ্নের সঠিক উত্তর **A** হয় তবে লিখতে হবে :
1. — A

- ভুল উত্তরের জন্য কোন নেগেটিভ মার্কিং নেই।

Turn Over

Paper Code : VI - A

Full Marks : 20 Time : Thirty Minutes

Choose the correct answer.

Each question carries 2 marks.

1. For any two events A and B,

(A) $P(A \bigcap B) \leq P(A) \leq P(A \bigcup B) \leq P(A) + P(B)$

(B) $P(A) \leq P(A \bigcap B) \leq P(A \bigcup B) \leq P(A) + P(B)$

(C) $P(A \bigcup B) \leq P(A \bigcap B) \leq P(A) \leq P(A) + P(B)$

(D) $P(A) + P(B) \leq P(A \bigcup B) \leq P(A \bigcap B) \leq P(A)$.

2. A function $f(x)$ given by $f(x) = \begin{cases} x & 0 < x < 1 \\ k - x & 1 < x < 2 \\ 0 & \text{elsewhere} \end{cases}$

is a probability density function for a suitable value of k. The value of k is

(A) 1

(B) 2

(C) 4

(D) $\frac{3}{2}$.

3. The random variable X is normal $(0, 1)$. The variance of e^X is

(A) $e - e^2$

(B) $e^3 - e$

(C) $e^2 - e$

(D) $e - e^3$.

4. The characteristic function of a $\gamma(l)$ variate is

(A) $(1 - it)^{2l}$

(B) $(1 - it)^{-2l}$

(C) $(1 - it)^{l}$

(D) $(1 - it)^{-l}$.

Turn Over

5. The acute angle θ between the least square regression lines is given by

(A) $\tan\theta = \frac{1-\rho^2}{\rho} \cdot \frac{\sigma_x \sigma_y}{\sigma_x^2+\sigma_y^2}$

(B) $\tan\theta = \frac{1-\rho^2}{\rho} \cdot \frac{\sigma_x^2+\sigma_y^2}{\sigma_x \sigma_y}$

(C) $\tan\theta = \frac{\rho}{1-\rho^2} \cdot \frac{\sigma_x \sigma_y}{\sigma_x^2+\sigma_y^2}$

(D) $\tan\theta = \frac{\rho}{1-\rho^2} \cdot \frac{\sigma_x^2+\sigma_y^2}{\sigma_x \sigma_y}$.

6. The skewness in a binomial distribution will be zero if

(A) $p < \frac{1}{2}$

(B) $p = \frac{1}{2}$

(C) $p > \frac{1}{2}$

(D) $p < q$.

7. The binary form of the hexadecimal digit 'F' is

(A) 1001

(B) 1011

(C) 1111

(D) 1101.

8. The relation between the backward difference operator ∇ and the shift operator E is

(A) $E - 1 = \nabla$

(B) $1 - E = \nabla$

(C) $E^{-1} - 1 = \nabla$

(D) $\nabla = 1 - E^{-1}$.

9. The 'degree of precession' of Trapezoidal rule is

(A) 1

(B) 2

(C) 4

(D) 5.

Turn Over

10. The relation between the forward difference operator $\triangle$ and the differential operator $D(=\frac{d}{dx})$ of differential calculus

(A) $D = \frac{1}{h}\left(\triangle + \frac{\triangle^2}{2} + \frac{\triangle^3}{3} + \frac{\triangle^4}{4} + \cdots \infty\right)$

(B) $D = \frac{1}{h}\left(\triangle - \frac{\triangle^2}{2} + \frac{\triangle^3}{3} - \frac{\triangle^4}{4} + \cdots \infty\right)$

(C) $D = \frac{1}{h}\left(\triangle + \frac{\triangle^3}{3} + \frac{\triangle^5}{5} + \cdots \infty\right)$

(D) $D = \frac{1}{h}\left(\triangle - \frac{\triangle^3}{3} + \frac{\triangle^5}{5} - \cdots \infty\right)$.

Turn Over

2020

MATHEMATICS (Honours)

Paper Code : VI - B

(New Syllabus)

Full Marks : 80 Time : Three Hours Thirty Minutes

The figures in the margin indicate full marks.

Notations and symbols have their usual meanings.

Group - A

(40 Marks)

Answer question no. 1 and any *six* from the rest.

1. The joint probability density function of two random variables X and Y is given by

$$f(x,y) = \begin{cases} k(3x+y) & 1 \leq x \leq 3,\ 0 \leq y \leq 2 \\ 0 & \text{elsewhere.} \end{cases}$$

Find

(i) The value of k and

(ii) $P(X+Y<2)$.

Or

Prove that $P(\bigcup_{i=1}^{\infty} A_i) \leq \sum_{i=1}^{\infty} P(A_i)$. **4**

2. If p and q are independent variables each uniformly distributed over the interval $(-1,1)$, then find the probability that the equation $x^2 - 2px + q = 0$ has real roots. **6**

3. If X is a standard normal variate, then prove that $Y = X^2/2$ is a $\gamma(1/2)$ variate. **6**

4. Prove the Schwartz's inequality for expectations that $[E(XY)]^2 \leq E(X^2) \cdot E(Y^2)$ and hence deduce the range of the correlation coefficient between X and Y. **6**

5. State and prove Tchebyshev's inequality. **1+5**

6. For a Binomial (n,p) distribution, prove that

$$\mu_{k+1} = p(1-p)\{n\mu_{k-1} + \frac{d\mu_k}{dp}\}$$

and hence obtain γ_1. **6**

Turn Over

7. Obtain a test for the null hypothesis $H_0 : m = m_0$ against the alternate hypothesis $H_1 : m > m_0$ for a normal (m, σ) population when σ is known. **6**

8. For a normal (m, σ) population, prove that the statistic $\frac{nS^2}{\sigma^2}$ has χ^2 distribution with $n-1$ degrees of freedom, where S^2 is the sample variance of a random sample of size n drawn from the given population. **6**

9. A random variable X can take all non-negative integral values and $P(X = x) = p(1-p)^x$, $x = 0, 1, 2, 3, \ldots$ where $p (0 < p < 1)$ is a parameter. Find the maximum likelihood estimate of p on the basis of a random sample $x_1, x_2, \ldots, x_n$ of size n drawn from the population of X. Examine whether the estimate is consistent. **6**

10. The mean and variance of a sample of size 400 from a normal population are found to be 18.35 and 3.25 respectively. Given $P(Z > 1.96) = 0.025$, Z being a standard normal variate, find 95% confidence interval for the population mean. **6**

Group-B

(40 Marks)

Answer question no. 11 and any *six* from the rest.

11. **(a)** Find the interpolation polynomial which corresponds to the following data:

x	-1	0	2	5
$f(x)$	9	5	3	15

4

or

(b) Write a short note on 'if' statement and 'if-else' statement. **4**

12. Show that the remainder in approximating $f(x)$ by the interpolation polynomial using distinct interpolating points $x_0, x_1, x_2, \cdots, x_n$ is of the form $(x - x_0)(x - x_1)\cdots(x - x_n)\frac{f^{n+1}(\xi)}{(n+1)!}$, where ξ lies between the minimum and maximum of the points $x_0, x_1, x_2, \ldots, x_n$. **6**

13. Establish Weddle's rule for numerical integration in composite form. State the error term. **5+1**

14. Write a program in $C/FORTRAN$ to compute the sum of the convergent series $x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots \infty$. **6**

15. Write a computer program in $C/FORTRAN$ to obtain the sum of matrices $[a_{ij}]_{m\times n}$ and $[b_{ij}]_{m\times n}$. **6**

16. Establish Langrange's polynomial interpolation formula with remainder term. **6**

17. Describe Gauss-Siedel method for numerical solution of a system of linear equations. State the condition of convergence of the method. **5+1**

Turn Over

18. Describe Newton-Raphson method for computing a simple real root of the equation $f(x) = 0$. Calculate the order of convergence of the method. **2+4**

19. Write a computer program in $C/FORTRAN$ to find mean and variance of a sample $(x_1, x_2, \cdots, x_n)$ of size n. **6**

20. If N denotes the total number of operations of multiplications and divisions required in Gauss elimination method, then prove that $N = \frac{n^3}{3} + n^2 - \frac{n}{3}$. **6**
