2020

MATHEMATICS (Honours)

Paper Code : VII - A & B

(New Syllabus)

Full Marks : 100

Time : Four Hours

Important Instructions for Multiple Choice Question (MCQ)

- Write Subject Name and Code, Registration number, Session and Roll number in the space provided on the Answer Script.

  Example : Such as for Paper III-A (MCQ) and III-B (Descriptive).

  Subject Code :

III	A	&	B

  Subject Name :

- Candidates are required to attempt all questions (MCQ). Below each question, four alternatives are given [i.e. (A), (B), (C), (D)]. Only one of these alternatives is 'CORRECT' answer. The candidate has to write the Correct Alternative [i.e. (A)/(B)/(C)/(D)] against each Question No. in the Answer Script.

  Example — If alternative A of 1 is correct, then write :
  1. — A

- There is no negative marking for wrong answer.

মাল্টিপল চয়েস প্রশ্নের (MCQ) জন্য জরুরী নির্দেশাবলী

- উত্তরপত্রে নির্দেশিত স্থানে বিষয়ের (Subject) নাম এবং কোড, রেজিস্ট্রেশন নম্বর, সেশন এবং রোল নম্বর লিখতে হবে।

উদাহরণ — যেমন Paper III-A (MCQ) এবং III-B (Descriptive)।

Subject Code : | III | A | & | B |

Subject Name : [ ]

- পরীক্ষার্থীদের সবগুলি প্রশ্নের (MCQ) উত্তর দিতে হবে। প্রতিটি প্রশ্নে চারটি করে সম্ভাব্য উত্তর, যথাক্রমে (A), (B), (C) এবং (D) করে দেওয়া আছে। পরীক্ষার্থীকে তার উত্তরের স্বপক্ষে (A) / (B) / (C) / (D) সঠিক বিকল্পটিকে প্রশ্ন নম্বর উল্লেখসহ উত্তরপত্রে লিখতে হবে।

উদাহরণ — যদি **1** নম্বর প্রশ্নের সঠিক উত্তর **A** হয় তবে লিখতে হবে :

1. — A

- ভুল উত্তরের জন্য কোন নেগেটিভ মার্কিং নেই।

Turn Over

Paper Code : VII - A

Full Marks : 20 | Time : Thirty Minutes

Choose the correct answer.

Each question carries 2 marks.

1. What is the total work done of moving a particle in a force field given by $\mathbf{F} = 3xy\,\hat{\mathbf{i}} - 5z\,\hat{\mathbf{j}} + 10x\,\hat{\mathbf{k}}$ along the curve $x = t^2 + 1$, $y = 2t^2$, $z = t^3$ from $t = 1$ to $t = 2$?

   (A) 230

   (B) 330

   (C) 303

   (D) 203

2. If S is a closed surface and V is the volume enclosed by S, then the value of $\iint\limits_S \mathbf{r}.\mathbf{n}\,dS$ is

   (A) $2V$

   (B) $3V$

   (C) V

   (D) $4V$

3. If O be the origin and $P = (1, 1)$, then the value of $\int\limits_{OP} \mathbf{F}.\,d\mathbf{r}$, where $\mathbf{F} = x\,\hat{\mathbf{i}} - y\,\hat{\mathbf{j}}$ is

   (A) 0

   (B) 1

   (C) 2

   (D) $\frac{1}{2}$

4. Let r be the distance of the point from the centre. The centre of gravity of a semi circular plate of radius a whose mass per unit area at any point varies as $\sqrt{a^2 - r^2}$ is

   (A) $(0, a)$

   (B) $(0, \frac{3a}{8})$

   (C) $(0, \frac{3a}{4})$

   (D) $(0, \frac{4a}{3})$

Turn Over

5. A circular area with radius a is immersed in a liquid with its centre at a depth h below the surface. Then the dept of centre of pressure below the free surface is -

(A) $h + \frac{a^2}{2h}$

(B) $h - \frac{a^2}{2h}$

(C) $h - \frac{a^2}{4h}$

(D) $h + \frac{a^2}{4h}$

6. The moment of inertia of a circular disc of radius a and of mass M about a diameter is -

(A) $\frac{Ma^2}{4}$

(B) $\frac{Ma^2}{2}$

(C) $\frac{Ma^2}{3}$

(D) $\frac{4Ma^2}{3}$

7. A force acting on a body where R is the normal reaction, $\mathbf{F}$ is the frictional force and μ is the coefficient of friction. Then the body will roll if -

(A) $\mu > \frac{\mathbf{F}}{R}$

(B) $\mu < \frac{\mathbf{F}}{R}$

(C) $\mu = \frac{\mathbf{F}}{R}$

(D) $\mu \neq \frac{\mathbf{F}}{R}$

8. A uniform beam of thickness $2b$ rests symmetrically on a perfectly rough horizontal cylinder of radius a. Then the equilibrium of the beam will be stable or unstable according as

(A) $b < a$ or $b > a$

(B) $b = a$ or $b = 2a$

(C) $b > a$ or $b < a$

(D) none of these

9. The centre of gravity of the arc of the astroid $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$ lying in the first quadrant is -

(A) (a, a)

(B) $(\frac{2a}{3}, \frac{2a}{3})$

(C) $(\frac{2a}{5}, \frac{2a}{5})$

(D) $(\frac{5a}{2}, \frac{5a}{2})$

Turn Over

10. If k be the radius of gyration of a rigid body of mass M about the axis, then the kinetic energy of the rigid body rotating with constant angular velocity ω about the axis is -

(A) $\frac{1}{2}Mk^2\omega^2$

(B) $\frac{1}{2}Mk^2\omega$

(C) $Mk^2\omega^2$

(D) $Mk^2\omega$

Turn Over

P-III/(1+1+1)/H/20(N)

2020

MATHEMATICS (Honours)

Paper Code : VII - B

(New Syllabus)

Full Marks : 80 Time : Three Hours Thirty Minutes

The figures in the margin indicate full marks.

Notations and symbols have their usual meanings.

Group - A

(10 Marks)

Answer any *two* questions : 5×2=10

1. Let C be the triangle formed by the points $(0,0)$, $\left(\frac{\pi}{2},0\right)$ and $\left(\frac{\pi}{2},1\right)$. Evaluate $\oint_C \{(y-\sin x)\,dx + \cos x\,dy\}$ directly and by Green's theorem. [5]

2. Verify Stokes' theorem for $\vec{F} = xz\,\hat{\mathbf{i}} - y\,\hat{\mathbf{j}} + x^2y\,\hat{\mathbf{k}}$, where S is the surface of the region bounded by $x=0$, $y=0$, $z=0$, $2x+y+2z=8$ which is not included in the xz - plane. [5]

3. Let C be the boundary of the region enclosed by $y=x^2$, $y=x+2$ and $x=0$. Then find the value of the line integral $\oint_C \{(xy-y^2)dx - x^3dy\}$ where C is traversed in the counter clockwise direction. [5]

4. Evaluate $\int_C \vec{F}.d\vec{r}$, where $\vec{F}=(x-y, x+y)$ and C is the curve formed by joining the arcs of $y=x^2$, $x=y^2$. [5]

Group-B

(25 Marks)

Answer question no. 5 and any three from the rest.

5. Answer any one question: $[4\times 1 = 4]$

   (a) A heavy body is in limiting equilibrium on a rough inclined plane under the action of gravity only. Prove that the inclination of the plane is equal to the angle of friction.

   (b) The moments of a system of forces about the points $(0,0)$, $(a,0)$ and $(0,a)$ are aw, $2aw$ and $3aw$, respectively. Find the components of their resultant parallel to the rectangular coordinate axes.

6. Three forces P, Q, R act along the sides of a triangle formed by the lines $x+y=3$, $2x+y=1$ and $x-y+1=0$. Find the equation of the line of action of the resultant. [7]

Turn Over

7. A body rests in equilibrium on another fixed body, there being enough friction to prevent sliding. The portions of the two bodies in contact are spherical and of radii r and R and the line of joining their centres in position of equilibrium is vertical. Show that the equilibrium is stable provided $\frac{1}{h} > \frac{1}{r} + \frac{1}{R}$, where h is the height of the centre of gravity of the upper body in position of equilibrium above the point of contact. [7]

8. A square frame $ABCD$ formed by four equal jointed rods hangs from A, the shape being maintained by a string joining mid points of AB, BC. Prove that the ratio of the tension of the string to the reaction at C is $\frac{8}{\sqrt{5}}$. [7]

9. A solid frustum of paraboloid of revolution of height h and latus rectum $4a$, rests, with its vertex on the vertex of a paraboloid of revolution, whose latus rectum is $4b$. Show that equilibrium is stable if $h < \frac{3ab}{a+b}$. [7]

10. A perfectly rough plane is inclined at an angle α to the horizon; show that the least eccentricity of the ellipse which can rest on the plane is $\sqrt{\frac{2\sin\alpha}{1+\sin\alpha}}$. [7]

Group-C

(25 Marks)

Answer question no. 11 and any three from the rest.

11. Answer any one question : [$4 \times 1 = 4$]

(a) (i) State the principal of conservation of angular momentum.

(ii) State the D' Alembert's principle.

[2+2=4]

(b) Prove that the equation of the momental ellipsoid at the centre of the elliptic plate is

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \left(\frac{1}{a^2} + \frac{1}{b^2}\right) z^2 = \text{constant}.$$

[4]

12. A rod of length $2a$, is suspended by a string of length ℓ, attached to one of its ends. The string and the rod revolve with angular velocity about the vertical through that end of the string and their inclinations to the vertical are θ and ϕ respectively, then show that $\frac{3\ell}{a} = \frac{(4\tan\theta - 3\tan\phi)\sin\phi}{(\tan\phi - \tan\theta)\sin\theta}$. [7]

Turn Over

13. Show that the Kinetic energy (K.E.) of a rigid body of mass M, is given by

$$K.E. = \frac{1}{2}Mv^2 + \frac{1}{2}MK^2\dot{\theta}^2,$$

where v and K are velocity of the centre of inertia and radius of gyration about a line through centre of gravity, respectively; and θ has its usual meaning. [7]

14. A solid homogeneous cone of height h and semi vertical angle α oscillates about a diameter of its base. Show that the length of simple equivalent pendulum is $\dfrac{(2+3\tan^2\alpha)\,h}{5}$. [7]

15. A plank of mass M, is initially at rest along a line of greatest slope of a smooth plane inclined at an angle α to the horizon, and a man , of mass M', starting from the upper end walks down the plank so that it does not move, show that he gets to the other end in time $\sqrt{\dfrac{2M'a}{(M+M')\,g\sin\alpha}}$, where a is the length of the plank. [7]

16. A thin heavy rectangular disc can turn freely about an axis in its own plane, and this axis revolves horizontally with a uniform angular velocity ω about the fixed point on itself. Show that the inclination θ of the plane of the disc to the vertical is $\cos^{-1}\left(\dfrac{gh}{k^2\omega^2}\right)$, where h is the distance of the centre of inertia of the rectangular disc from the axis and k is the radius of gyration of the disc about the axis. [7]

Group-D

(20 Marks)

Answer question no. 17 and any two from the rest.

17. Answer any one question : $[6 \times 1 = 6]$

(a) If X, Y, Z be the external force components per unit mass of a homogeneous, incompressible fluid at rest at the point (x, y, z), show that the differential equations of the surfaces of equal pressure and equal density are

$$\frac{dx}{\frac{\partial Z}{\partial y} - \frac{\partial Y}{\partial z}} = \frac{dy}{\frac{\partial X}{\partial z} - \frac{\partial Z}{\partial x}} = \frac{dz}{\frac{\partial Y}{\partial x} - \frac{\partial X}{\partial y}}.$$

(b) A mass of liquid is in equilibrium under the action of conservative system of forces. Prove that the surface of equal pressure and the surface of equal density coincide.

18. Show that the depth of the centre of pressure of a trapezium of which one side of length a is in the surface and the parallel side of length b at a depth c is $\dfrac{(a+3b)}{(a+2b)} \cdot \dfrac{c}{2}$. [7]

Turn Over

19. If near the earth's surface gravity be assumed to be constant and the temperature in the atmosphere at a height z to be given by $t = t_0\left(1 - \frac{z}{nH}\right)$, where H is the height of the homogeneous atmosphere, show the pressure in the atmosphere will be given by the equation $p = p_0\left(1 - \frac{z}{nH}\right)^n$. [7]

20. In a vertical column of perfect gas the pressure and absolute temperature at any height z are p and T. Prove that $z = \frac{p_0}{\rho_0 g T_0}\int_p^{p_0} \frac{T}{p}dp$, where p_0, ρ_0, T_0 are pressure, density and absolute temperature at the bottom and g has its usual meaning. [7]

21. Show that the general equation of equilibrium of a fluid revolving uniformly about an axis is given by

$$dp = \rho\left(X\ dx + Y\ dy + Z\ dz + \omega^2 r\cos\theta\ dx + \omega^2 r\sin\theta\ dy\right),$$

where the symbols have their usual meanings. [7]