

2020

MATHEMATICS (Honours)

Paper Code : III - A & B

[New Syllabus]

Important Instructions for Multiple Choice Question (MCQ)

- Write Subject Name and Code, Registration number, Session and Roll number in the space provided on the Answer Script.

Example : Such as for Paper III-A (MCQ) and III-B (Descriptive).

Subject Code :

III	A	&	B
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Subject Name :

- Candidates are required to attempt all questions (MCQ). Below each question, four alternatives are given [i.e. (A), (B), (C), (D)]. Only one of these alternatives is 'CORRECT' answer. The candidate has to write the Correct Alternative [i.e. (A)/(B)/(C)/(D)] against each Question No. in the Answer Script.

Example — If alternative A of 1 is correct, then write :

1. — A

- There is no negative marking for wrong answer.

মাল্টিপল চয়েস প্রশ্নের (MCQ) জন্য জরুরী নির্দেশাবলী

- উত্তরপত্রে নির্দেশিত স্থানে বিষয়ের (Subject) নাম এবং কোড, রেজিস্ট্রেশন নম্বর, সেশন এবং রোল নম্বর লিখতে হবে।

উদাহরণ — যেমন Paper III-A (MCQ) এবং III-B (Descriptive)।

Subject Code :

III	A	&	B
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Subject Name :

- পরীক্ষার্থীদের সবগুলি প্রশ্নের (MCQ) উত্তর দিতে হবে। প্রতিটি প্রশ্নে চারটি করে সম্ভাব্য উত্তর, যথাক্রমে (A), (B), (C) এবং (D) করে দেওয়া আছে। পরীক্ষার্থীকে তার উত্তরের স্বপক্ষে (A)/(B)/(C)/(D) সঠিক বিকল্পটিকে প্রশ্ন নম্বর উল্লেখসহ উত্তরপত্রে লিখতে হবে।

উদাহরণ — যদি 1 নম্বর প্রশ্নের সঠিক উত্তর A হয় তবে লিখতে হবে :

1. – A

- ভুল উত্তরের জন্য কোন নেগেটিভ মার্কিং নেই।

Paper Code : III - A

Full Marks : 20

Time : Thirty Minutes

Choose the correct answer.

Answer *all* the following questions,
each question carries 2 marks.

Notations and symbols have their usual meanings.

1. The point on $\frac{l}{r} = 1 - \cos \theta$ which has the smallest radius vector is
 - (A) $(\frac{l}{2}, \pi)$
 - (B) (l, π)
 - (C) $(l, \frac{\pi}{2})$
 - (D) $(\frac{l}{2}, -\pi)$.
2. The polar of the point $(0, 0)$ w.r.t. $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ is
 - (A) $-gx + fy = 0$
 - (B) $gx + fy = 0$
 - (C) $gx + fy + c = 0$
 - (D) $gx + fy - c = 0$.
3. The equation $6x^2 - 5xy - 6y^2 + 14x + 5y + 4 = 0$ represents
 - (A) an ellipse
 - (B) a hyperbola
 - (C) a pair of straight lines
 - (D) a parabola.
4. The foot of the perpendicular from the origin to the plane is $(3, 2, -1)$. The equation of the plane is
 - (A) $3x + 2y + z = 14$
 - (B) $3x + 2y + z + 14 = 0$
 - (C) $3x - 2y + z = 14$
 - (D) $3x + 2y - z = 14$.

5. The volume of the tetrahedron formed by the points $(0, 0, 0)$, $(3, 2, -1)$, $(4, 1, 4)$ and $(5, 3, 5)$ is
- (A) $\frac{14}{3}$ cubic unit
 (B) $\frac{14}{5}$ cubic unit
 (C) $\frac{14}{11}$ cubic unit
 (D) $\frac{14}{13}$ cubic unit.
6. If a right circular cone has three mutually perpendicular generators, then its semi-vertical angle is given by
- (A) $\tan^{-1}(\sqrt{3})$
 (B) $\tan^{-1}(\sqrt{2})$
 (C) $\tan^{-1}(\sqrt{5})$
 (D) $\tan^{-1}(\sqrt{7})$.
7. The velocity of a particle moving in a straight line at any instant t , when its distance from the origin is x , is given by $x = \frac{1}{2}v^2$. The acceleration of the particle is
- (A) 2 unit
 (B) -2 unit
 (C) 1 unit
 (D) 3 unit.
8. A particle describes the curve $r \cosh(n\theta) = a$ under a force F to the pole, the law of force is proportional to
- (A) $1/r$
 (B) $1/r^2$
 (C) $1/r^3$
 (D) none of the above.
9. A particle coming rest from infinity will reach the earth's surface with a velocity
- (A) \sqrt{gr}
 (B) $\sqrt{2gr}$
 (C) $\sqrt{3gr}$
 (D) $2\sqrt{gr}$.

10. A particle describes a parabola with uniform speed. The angular velocity of the particle about the focus S , at any point P , varies inversely as

(A) $(SP)^{5/2}$

(B) $(SP)^{7/2}$

(C) $(SP)^{3/2}$

(D) $(SP)^{9/2}$.



2020

MATHEMATICS (Honours)

Paper Code : III - B

[New Syllabus]

Full Marks : 80

Time : Three Hours Thirty Minutes

The figures in the margin indicate full marks.

Notations and symbols have their usual meanings.

Group-A (20 Marks)

Answer any **four** questions.

5 × 4 = 20

1. If the equation $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represents two straight lines equidistant from the origin, then show that $f^4 - g^4 = c(bf^2 - ag^2)$.
2. Reduce the equation $11x^2 + 4xy + 14y^2 - 26x - 32y + 23 = 0$ to the canonical form and state the nature of the conic.
3. Prove that the two conics $\frac{l_1}{r} = 1 - e_1 \cos \theta$ and $\frac{l_2}{r} = 1 - e_2 \cos(\theta - \alpha)$ will touch one another, if $l_1^2(1 - e_2^2) + l_2^2(1 - e_1^2) = 2l_1l_2(1 - e_1e_2 \cos \alpha)$.
4. Find the locus of the poles with respect to the circle $x^2 + y^2 = a^2$ of the tangents to the circle $x^2 + y^2 = 2ax$.
5. Show that the locus of the middle points of the normal chords of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is $\left(\frac{x^2}{a^2} + \frac{y^2}{b^2}\right)^2 \cdot \left(\frac{a^6}{x^2} + \frac{b^6}{y^2}\right) = (a^2 - b^2)^2$.
6. The origin is shifted to the point $(3, -1)$ and the axes are rotated through an angle $\tan^{-1} \frac{3}{4}$. If the co-ordinates of a point are $(5, 10)$ in the new system, then find the co-ordinates in the old system.

Group-B
(25 Marks)

Answer any **five** questions.

5 × 5 = 25

7. A point P moves on the plane $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$, which is fixed and the plane through P perpendicular to OP meets the axes in A, B, C . If the planes through A, B, C parallel to co-ordinate planes meet in a point Q , then show that the locus of Q is

$$\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = \frac{1}{ax} + \frac{1}{by} + \frac{1}{cz}.$$

8. Show that the locus of the variable line which intersects the three lines $y = mx, z = c; y = -mx, z = -c; y = z, mx = -c$ is the surface $y^2 - m^2x^2 = z^2 - c^2$.
9. A variable sphere passes through the points $(0, 0, \pm c)$ and cuts the straight lines $y = x \tan \alpha, z = c; y = -x \tan \alpha, z = -c$ in the points P, P' . If $PP' = 2a$, a constant, then show that the centre of the sphere lies on the circle $z = 0, x^2 + y^2 = (a^2 - c^2) \operatorname{cosec}^2 2\alpha$.
10. Show that the angle between the lines of section of the plane $x + y + z = 0$ and the cone $\frac{yz}{b-c} + \frac{zx}{c-a} + \frac{xy}{a-b} = 0$ is 60° .
11. The section of the cone whose guiding curve is the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, z = 0$ by the plane $x = 0$ is a rectangular hyperbola. Show that the locus of the vertex is the surface $\frac{x^2}{a^2} + \frac{y^2+z^2}{b^2} = 1$.
12. Show that the equation of the right circular cylinder, whose guiding curve is the circle through the points $(1, 0, 0), (0, 1, 0)$ and $(0, 0, 1)$ is $x^2 + y^2 + z^2 - xy - yz - zx = 1$.
13. Find the locus of a luminous point, if the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ casts a circular shadow on the plane $z = 0$.
14. Show that the feet of the normals from the point (α, β, γ) to the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ lie on the curve of intersection of the ellipsoid and the cone

$$\frac{\alpha a^2(b^2 - c^2)}{x} + \frac{\beta b^2(c^2 - a^2)}{y} + \frac{\gamma c^2(a^2 - b^2)}{z} = 0.$$

Group-C
(35 Marks)

Answer any **five** questions.

7 × 5 = 35

15. A particle moves from rest in a straight line under an attractive force $\mu \times (\text{distance})^{-2}$ per unit mass to a fixed point on the line. Show that if the initial distance from the centre of force is $2a$, then the distance will be a after a time $\left(\frac{\pi}{2} + 1\right) \left(\frac{a^3}{\mu}\right)^{1/2}$.

16. Three elastic balls of masses m_1, m_2, m_3 , lie in a straight line on a horizontal table and m_1 is projected towards m_2 . If the velocity of m_1 after striking m_2 is equal to that of m_2 after striking m_3 , then prove that

$$(m_1 + m_2)(m_2 + m_3)e = m_1 m_3 (1 + e)^2.$$

17. Prove that the velocity and acceleration components referred to rotating axes are respectively

$$\dot{x} - y\dot{\theta}, \quad \dot{y} + x\dot{\theta}, \quad \ddot{x} - x\dot{\theta}^2 - 2y\ddot{\theta} - y\ddot{\theta}, \quad \ddot{y} - y\dot{\theta}^2 + 2x\ddot{\theta} + x\ddot{\theta}.$$

18. A particle moves under a force $m\mu\{3au^4 - 2(a^2 - b^2)u^5\}$, ($a > b$), and is projected from an apse at a distance $a + b$ with velocity $\frac{\sqrt{\mu}}{a+b}$. Show that its orbit is $r = a + b \cos \theta$.

19. A particle descends down a rough circular tube starting from rest at an extremity of horizontal diameter. If it stops at the lowest point, then show that the coefficient of the friction satisfies the equation $3\mu e^{-\mu\pi} + 2\mu^2 - 1 = 0$.

20. A heavy particle falls from rest under gravity in a medium, the resistance of which varies as the square of the velocity. Show that the depth x described in time t is given by $x = \frac{V^2}{g} \log \cosh \frac{gt}{V}$, where V is the terminal velocity.

21. If a rocket, originally of mass M , throws off every unit of time a mass eM with relative velocity V and if M' is the mass of the case etc., then show that it cannot rise at once unless $eV > g$, not at all unless $\frac{eMV}{M'} > g$. If it just rises vertically at once, then show that the greatest velocity is $V \log \frac{M}{M'} - \frac{g}{e} \left(1 - \frac{M'}{M}\right)$.