RAIGANJ SURENDRANATH MAHAVIDYALAYA
DEPARTMENT OF MATHEMATICS
INTERNAL EXAMINATION, 2021

Date: 13.03.2021

B.Sc.(Hons) 3rd Semester (DC-07)

Full Marks: 18

Time: One hour

Group-A

Answer all the following questions: $4 \times 2 = 8$

1. Show that the function $f(x,y) = \begin{cases} \dfrac{x^3+y^3}{x-y}, & x \neq y \\ 0, & x = y \end{cases}$ is discontinuous at $(0,0)$.

2. If $V = \sin^{-1}\sqrt{\dfrac{x^{1/3}+y^{1/3}}{x^{1/2}+y^{1/2}}}$, then prove that $x\dfrac{\partial V}{\partial x} + y\dfrac{\partial V}{\partial y} + \dfrac{1}{12}\tan V = 0$.

3. Find $\operatorname{div}\vec{F}$ and $\operatorname{Curl}\vec{F}$, where $\vec{F} = \operatorname{grad}(x^3+y^3+z^3-3xyz)$.

4. If $u = \dfrac{x+y}{1-xy}$, $v = \tan^{-1}x + \tan^{-1}y$. Find $\dfrac{\partial(u,v)}{\partial(x,y)}$. Find the relation between them.

Group-B

Answer all the questions: $5 \times 2 = 10$

5. Show that $\displaystyle\iiint_E \frac{dx\,dy\,dz}{x^2+y^2+(z-2)^2} = \pi\left(2 - \frac{3}{2}\log 3\right)$, where E is the region bounded by the sphere $x^2+y^2+z^2 = 1$. $\boxed{5}$

6. Define 'Green's Theorem' on Vector Calculus.
Using Green's theorem, show that the area bounded by a simple closed curve C is given by $\frac{1}{2}\oint_C (x\,dy - y\,dx)$.
Hence find the area of the ellipse $x = a\cos\theta$, $y = b\sin\theta$, $(0 \leq \theta \leq 2\pi)$.

$\boxed{1+2+2=5}$

—— End ——