2020

MATHEMATICS (Honours)

Paper Code : V-A & B

(New Syllabus)

Full Marks : 100

Time : Four Hours

Important Instructions for Multiple Choice Question (MCQ)

- Write Subject Name and Code, Registration number, Session and Roll number in the space provided on the Answer Script.

  Example : Such as for Paper III-A (MCQ) and III-B (Descriptive).

  Subject Code :

III	A	&	B

  Subject Name :

- Candidates are required to attempt all questions (MCQ). Below each question, four alternatives are given [i.e. (A), (B), (C), (D)]. Only one of these alternatives is 'CORRECT' answer. The candidate has to write the Correct Alternative [i.e. (A)/(B)/(C)/(D)] against each Question No. in the Answer Script.

  Example — If alternative A of 1 is correct, then write :
  1. — A

- There is no negative marking for wrong answer.

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মাল্টিপল চয়েস প্রশ্নের (MCQ) জন্য জরুরী নির্দেশাবলী

- উত্তরপত্রে নির্দেশিত স্থানে বিষয়ের (Subject) নাম এবং কোড, রেজিস্ট্রেশন নম্বর, সেশন এবং রোল নম্বর লিখতে হবে।

  উদাহরণ — যেমন Paper III-A (MCQ) এবং III-B (Descriptive)।

  Subject Code : | III | A | & | B |

  Subject Name : [   ]

- পরীক্ষার্থীদের সবগুলি প্রশ্নের (MCQ) উত্তর দিতে হবে। প্রতিটি প্রশ্নে চারটি করে সম্ভাব্য উত্তর, যথাক্রমে (A), (B), (C) এবং (D) করে দেওয়া আছে। পরীক্ষার্থীকে তার উত্তরের স্বপক্ষে (A) / (B) / (C) / (D) সঠিক বিকল্পটিকে প্রশ্ন নম্বর উল্লেখসহ উত্তরপত্রে লিখতে হবে।

  উদাহরণ — যদি **1** নম্বর প্রশ্নের সঠিক উত্তর **A** হয় তবে লিখতে হবে :
  1. — A

- ভুল উত্তরের জন্য কোন নেগেটিভ মার্কিং নেই।

Turn Over

Paper Code : V - A

Full Marks : 20 Time : Thirty Minutes

Choose the correct answer.
Each question carries 2 marks.

1. Let $f:[0,1]\to\mathbb{R}$ be such that

$$f(x)=\begin{cases}\frac{1}{n} & \text{if } \quad x=\frac{1}{n}, n\in\mathbb{N}\\ 0 & \text{otherwise.}\end{cases}$$

Then the value of $\int_0^1 f(x)dx$

(A) is -1

(B) is 1

(C) is 0

(D) does not exist

2. The radius of convergence of the power series $\sum_{n=0}^{\infty}\frac{x^n}{n!}$ is

(A) 1

(B) 0

(C) $\frac{1}{2}$

(D) ∞

3. Define $f:[0,2]\longrightarrow\mathbb{R}$ by

$$f(x)=\begin{cases}1 & \text{if} \quad x=1\\ 0 & \text{if} \quad x\neq 1\end{cases}$$

and define $g:[0,1]\longrightarrow\mathbb{R}$ by

$$g(x)=\begin{cases}1 & \text{if} \quad x \quad \text{is rational}\\ 0 & \text{if} \quad x \quad \text{is irrational}\end{cases}$$

Then

(A) both *f*, *g* are Riemann integrable

(B) *f* is Riemann integrable but *g* is not Riemann integrable

(C) *g* is Riemann integrable but *f* is not Riemann integrable

(D) None of these

Turn Over

4. Let $x; y \in \mathbb{R}$ and define

$$d_1(x, y) = (x-y)^2 \text{ and } d_2(x, y) = |x^2 - y^2|$$

Then

(A) both d_1 and d_2 are metric

(B) d_1 is a metric but d_2 is not a metric

(C) d_1 is not a metric but d_2 is a metric

(D) neither d_1 and d_2 is a metric

5. If $f(z) = u + iv$ in polar form is analytic then $\frac{\partial u}{\partial r}$ is

(A) $\frac{\partial v}{\partial \theta}$

(B) $r\frac{\partial v}{\partial \theta}$

(C) $\frac{1}{r}\frac{\partial v}{\partial \theta}$

(D) $-\frac{\partial v}{\partial \theta}$

6. If A is an open set and B is a closed set in a metric space $(X; d)$, then

(A) $A \setminus B$ is open and $B \setminus A$ is closed

(B) $A \setminus B$ is closed and $B \setminus A$ is open

(C) $A \setminus B$ is open and $B \setminus A$ is open

(D) $A \setminus B$ is closed and $B \setminus A$ is closed.

7. If $I_n = (-\frac{n}{n+1}; \frac{n}{n+1})$, for $n = 1, 2, 3, \ldots$ then $\bigcap_{n=1}^{\infty}(-\frac{n}{n+1}; \frac{n}{n+1})$ equal to

(A) $(-1, 1)$

(B) $\left[-\frac{1}{2}, \frac{1}{2}\right]$

(C) $\left(-\frac{1}{2}, \frac{1}{2}\right)$

(D) $\{0\}$

Turn Over

8. $\lim_{x \to 0} \frac{x}{1-e^{x^2}} \int_0^x e^{t^2} dt$

(A) = 0

(B) = 1

(C) = – 1

(D) does not exist

9. The maximum of $f(x, y) = x^2 + 2y^2$ on the circle $x^2 + y^2 = 1$ is —

(A) 0

(B) 1

(C) 2

(D) 3

10. The value of the integral $\iint_R xy(x+y)dxdy$, where R = [0, 1]× [0,1], is

(A) 3

(B) $\frac{1}{3}$

(C) 0

(D) 1

Turn Over

2020

MATHEMATICS (Honours)

Paper Code : V - B

(New Syllabus)

Full Marks : 80 Time : Three Hours Thirty Minutes

The figures in the margin indicate full marks.

Notations and symbols have their usual meanings.

Group - A

(50 Marks)

Answer any *five* questions : 10×5=50

1. (a) Define a compact set. Show that a compact subset of real numbers must be closed and bounded.

   (b) Prove that the function $f(x) = \sum_{n=1}^{\infty} \dfrac{\sin(nx^2)}{1+n^3}$, $x \in \mathbb{R}$ is continuously differentiable on $\mathbb{R}$.

2. (a) Find the points on the circle $x^2 + y^2 = 80$ which are closest to and farthest from the point (1,2). [5]

   (b) Show that the sequence $\{f_n\}$ of functions, where $f_n(x) = nxe^{-nx^2}$ is pointwise convergent but not uniformly convergent on $[0,\infty)$. [5]

3. (a) Show that a necessary and sufficient condition for a bounded real valued function f to be Riemann integrable on $[a,b]$ is that for each $\varepsilon > 0$, $\exists\ \delta > 0$ such that $0 \le U(P,f) - L(P,f) < \varepsilon$ for every partition P with $\| P \| < \delta$. [6]

   (b) Show that $\left| \int_p^q \frac{\sin x}{x} dx \right| \le \frac{2}{p}$, if $q > p > 0$. [4]

4. (a) Show that the improper integral $\int\limits_0^\infty \frac{\sin x}{x} dx$ is not absolutely convergent. [5]

   (b) By changing the order of integration prove that

$$\int_0^1 dx \int_0^{\frac{1}{x}} \frac{y^2 dy}{(x+y)^2\sqrt{1+y^2}} = \frac{1}{2}(2\sqrt{2}-1).$$

[5]

5. State and prove the Cantor's intersection theorem. Illustrate the theorem with an example. [2+6+2]

Turn Over

6. (a) Using differentiation under the sign of integral show that

$$\int_0^{\pi} \frac{\log(1 + a\cos x)}{\cos x} dx = \pi \sin^{-1} a,$$

for $|a| < 1$. [5]

(b) State and prove Taylor's theorem for the function of two variables. [1+4]

7. (a) Test the convergence of beta and gamma functions. [3+4]

(b) If $\int_1^e \frac{dt}{t} = 1$, then show that $2 < e < 3$. [3]

8. (a) Evaluate $\iint_R f(x, y)dxdy$ over the rectangle $R = [0, 1] \times [0, 1]$ where $f(x, y) = x + y$ if $x^2 < y < 2x^2$ and $f(x, y) = 0$ elsewhere. [5]

(b) If a series $\sum f_n$ converges uniformly to f on $[a, b]$ and each term f_n is integrable on $[a, b]$, then show that f is integrable on $[a, b]$. [5]

Group-B
(15 Marks)

9. Answer any three questions. $5 \times 3 = 15$

(a) Show that every metric space is a Hausdorff space.

(b) Let (X, d) be any metric space, then show that the function d_1 defined by $d_1(x, y) = \frac{d(x,y)}{1+d(x,y)}$, $\forall x, y \in X$ is a metric on X.

(c) Show that $C[a, b]$ is a separable metric space under 'sup' metric.

(d) For points x, y, a, b in a metric space (X, d), show that $|d(x, y) - d(a, b)| \leq d(x, a) + d(y, b)$.

(e) Let (X, d) be a metric space and $A \subseteq X$. Prove that $\text{Int}(A)$ is the largest open set contained in A.

Group-C
(15 Marks)

10. Answer any three questions. $5 \times 3 = 15$

(a) Prove that the function $f(z) = \frac{x^3(1+i) - y^3(1-i)}{x^2+y^2}$, when $z = x + iy \neq 0$ and $f(0) = 0$ is continuous.

Turn Over

(b) Let $f(z) = u(x, y) + iv(x, y)$, $z = x + iy$ and $z_0 = x_0 + iy_0$. Let the function $f(z)$ be defined in a domain D except possibly at the point z_0 in D. Then prove that $\lim_{z \to z_0} f(z) = w_0 = u_0 + iv_0$ if and only if $\lim_{z \to z_0} u(x, y) = u_0$ and $\lim_{z \to z_0} v(x, y) = v_0$.

(c) Show that the function $u = x^3 - 3xy^2$ is harmonic and also find the corresponding analytic function.

(d) Show that the real and imaginary parts of an analytic function satisfy Laplace's equation.

(e) Show that the function $f = u + iv$ where $u = x^2 + y^2$ and $v = xy$ is nowhere analytic.