H/20 (SEM-II)

2020

MATHEMATICS (Honours)

Paper Code: MTMH-H- DC03

[CBCS]

Full Marks: 18 Time: 1hr

Notations and symbols have their usual meanings.

Part-A.

Answer all the questions.
Each question carries 2 marks

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1. Find the supremum and infimum of the set

$$S = \{1, 1+\frac{1}{2}, 1+\frac{1}{2}+\frac{1}{2^2}, ..., 1+\frac{1}{2}+\frac{1}{2^2}+\frac{1}{2^3}....\infty\}.$$

2. Show that the set $S = \{1 + \frac{(-1)^n}{n} : n \in N\}$ is closed.

3. Show that the infinite series

$$\sum_{n=1}^{\infty} \frac{1}{2^n} = \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + ...$$

is convergent.

Part-B.

Answer any three questions.
Each question carries 4 marks

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4. Use Cauchy's general principle of convergence to prove that the sequence $\{U_n\}$, where $U_n = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + ... + \frac{1}{n}$ is not convergent.

5. Test the convergence of the series

$$\frac{1}{2} + \frac{1}{3} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{2^3} + \frac{1}{3^3} + ...$$

6. Show that the function $f(x) = \sin x, x \in R$ is uniformly continuous on R.

7. Let f be a function defined by

$$\begin{aligned} f(x) &= 1, if \quad x \text{ is rational} \\ &= -1, if \quad x \text{ is irrational.} \end{aligned}$$

Then show that f is discontinuous at every point of R.