H/20 (SEM-II)

2020

MATHEMATICS (Honours)

Paper Code: MTMH-H- DC04

[CBCS]

Full Marks: 18 Time: 1hr

Notations and symbols have their usual meanings.

Part-A.

Answer all the questions.
Each question carries 2 marks

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1. In a group $(G, \circ)$,'a' is an element of order 30. Find the order of a^{18}.

2. Let D be an integral domain and $a, b \in D$. If $a^5 = b^5$ and $a^8 = b^8$, then prove that $a = b$.

3. Prove that the polynomial $f(x) = x^3 + x + 1$ is irreducible over Z_2, where Z stands for set of all integer.

Part-B.

Answer any three questions.
Each question carries 4 marks

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4. If H be any sub group of a group G, then prove that H is normal in G iff $aHa^{-1} = H$ for all a in G.

5. If R be a commutative ring with unity, then prove that R is an integral domain iff for every non-zero element b in R, $b.p = b.q \Rightarrow p = q$, where $p, q \in R$.

6. Let $(G, \circ)$ be a group and $a, b \in G$. If $o(a) = 3$ and $a \circ b \circ a^{-1} = b^2$, then find $o(b)$ if $b \neq$identity element.

7. Prove that an ideal U of a ring R is maximal iff the quotient ring R/U is simple.