2020

MATHEMATICS (Honours)

Paper Code : I - A & B

[New Syllabus]

Important Instructions for Multiple Choice Question (MCQ)

- Write Subject Name and Code, Registration number, Session and Roll number in the space provided on the Answer Script.

  Example : Such as for Paper III-A (MCQ) and III-B (Descriptive).

  Subject Code :

III	A	&	B

  Subject Name :

- Candidates are required to attempt all questions (MCQ). Below each question, four alternatives are given [i.e. (A), (B), (C), (D)]. Only one of these alternatives is 'CORRECT' answer. The candidate has to write the Correct Alternative [i.e. (A)/(B)/(C)/(D)] against each Question No. in the Answer Script.

  Example — If alternative A of 1 is correct, then write :
  1. — A

- There is no negative marking for wrong answer.

মাল্টিপল চয়েস প্রশ্নের (MCQ) জন্য জরুরী নির্দেশাবলী

- উত্তরপত্রে নির্দেশিত স্থানে বিষয়ের (Subject) নাম এবং কোড, রেজিস্ট্রেশন নম্বর, সেশন এবং রোল নম্বর লিখতে হবে।

উদাহরণ — যেমন Paper III-A (MCQ) এবং III-B (Descriptive)।

Subject Code : | III | A | & | B |

Subject Name : [ ]

- পরীক্ষার্থীদের সবগুলি প্রশ্নের (MCQ) উত্তর দিতে হবে। প্রতিটি প্রশ্নে চারটি করে সম্ভাব্য উত্তর, যথাক্রমে (A), (B), (C) এবং (D) করে দেওয়া আছে। পরীক্ষার্থীকে তার উত্তরের স্বপক্ষে (A) / (B) / (C) / (D) সঠিক বিকল্পটিকে প্রশ্ন নম্বর উল্লেখসহ উত্তরপত্রে লিখতে হবে।

উদাহরণ — যদি **1** নম্বর প্রশ্নের সঠিক উত্তর **A** হয় তবে লিখতে হবে :

1. — A

- ভুল উত্তরের জন্য কোন নেগেটিভ মার্কিং নেই।

Paper Code : I - A

Full Marks : 20 Time : Thirty Minutes

Choose the correct answer.

Answer *all* the following questions,
each question carries 2 marks.

Notations and symbols have their usual meanings.

1. If the equation $x^n - nqx + (n-1)r = 0$ has a pair of equal roots, then

   A. $r^{n-1} = q^n$

   B. $q^n r = 1$

   C. $q^n + 1 = r^2$

   D. $r^n - 1 = q^2$.

2. If $a, b, c > 0$ and $a + b + c = 1$, then $(1+a)(1+b)(1+c)$ has

   A. maximum value 8

   B. minimum value 8

   C. maximum value $\frac{64}{27}$

   D. minimum value $\frac{64}{27}$.

3. The unit digit of 7^{100} is

   A. 1

   B. 3

   C. 5

   D. 7.

4. If $x + \frac{1}{x} = 2\cos\frac{\pi}{7}$, then the value of $x^7 + \frac{1}{x^7}$ is

A. -1

B. 1

C. 2

D. -2.

5. $\begin{vmatrix} 1 & 1+x & 1+x+x^2 \\ 1 & 1+y & 1+y+y^2 \\ 1 & 1+z & 1+z+z^2 \end{vmatrix}$ is equal to

A. $(z-y)(z-x)(y-x)$

B. $(x-y)(x-z)(y-z)$

C. $(x-y)^2(y-z)^2(z-x)^2$

D. $(x^2-y^2)(y^2-z^2)(z^2-x^2)$.

6. The number of different reflexive relations of a set of four elements is

A. 2^2

B. 2^4

C. 2^9

D. 2^{12}.

7. Which of the following matrices has the same row space as the matrix $\begin{bmatrix} 4 & 8 & 4 \\ 3 & 6 & 1 \\ 2 & 4 & 0 \end{bmatrix}$?

A. $\begin{bmatrix} 1 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

B. $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

C. $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

D. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$.

8. If for any two vectors α, β of an Euclidean space we have $\|\alpha + \beta\| = \|\alpha - \beta\|$, then

A. $\langle \alpha, \beta \rangle = 0$

B. $\langle \alpha, \beta \rangle = 1$

C. $\|\alpha\| = \|\beta\|$

D. None of the above.

9. If R is a ring such that $a^2 = a$ for all $a \in R$, then we have

A. R is not commutative

B. R is commutative

C. $a = 0$ for all $a \in R$

D. None of the above.

10. The total number of unit elements in the ring $(\mathbb{Z}_{10}, +, .)$ is

A. 4

B. 0

C. 5

D. 9.

2020

MATHEMATICS (Honours)

Paper Code : I - B

[New Syllabus]

Full Marks : 80 Time : Three Hours Thirty Minutes

The figures in the margin indicate full marks.

Notations and symbols have their usual meanings.

Group-A

(30 Marks)

Answer any **six** questions

. $5 \times 6 = 30$

1. If $a_1, a_2, \ldots, a_n$ are n positive rational numbers and $S = a_1 + a_2 + \cdots + a_n$, then prove that $\left(\frac{S}{a_1} - 1\right)^{a_1} \left(\frac{S}{a_2} - 1\right)^{a_2} \cdots \left(\frac{S}{a_n} - 1\right)^{a_n} \leq (n-1)^S$. [5]

2. If $\cos\alpha + \cos\beta + \cos\gamma = \sin\alpha + \sin\beta + \sin\gamma = 0$, then prove that $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = \sin^2\alpha + \sin^2\beta + \sin^2\gamma = \frac{3}{2}$. [5]

3. Prove that $x^n - 1 = (x^2 - 1) \prod_{k=1}^{\frac{n-2}{2}} \left[x^2 - 2x\cos\frac{2k\pi}{n} + 1\right]$, if n is an even positive integer. Deduce that $\sin\frac{\pi}{32}\sin\frac{2\pi}{32}\cdots\sin\frac{15\pi}{32} = \frac{1}{2^{13}}$. [5]

4. If $\log\sin(\theta + i\phi) = \alpha + i\beta$ where $\theta, \phi, \alpha, \beta$ are real, then prove that

   (a) $2\cos 2\theta = e^{2\phi} + e^{-2\phi} - 4e^{2\alpha}$ [2]

   (b) $\cos(\theta - \beta) = e^{2\phi}\cos(\theta + \beta)$. [3]

5. If p is a prime number, then prove that $(p-1)! + 1 \equiv 0 \pmod{p}$. [5]

6. Prove that for any positive integer n, $4.6^n + 5^{n+1} - 9$ is divisible by 2. [5]

7. If a, b, c and d are roots of the equation $x^4 + x^3 + 2x^2 + x + 1 = 0$, then find the value of

(a) $(a^2+4)(b^2+4)(c^2+4)(d^2+4)$ [3]

(b) $(a^3+1)(b^3+1)(c^3+1)(d^3+1)$. [2]

8. Find the special roots of the equation $x^{20} - 1 = 0$. Deduce that $\cos\frac{\pi}{10}, \cos\frac{3\pi}{10}, \cos\frac{7\pi}{10}$ and $\cos\frac{9\pi}{10}$ are roots of the equation $16y^4 - 20y^2 + 5 = 0$. [5]

9. If α, β, γ are roots of the equation $x^3 - 2x^2 + 3x - 1 = 0$, then find the equation whose roots are $\frac{\beta\gamma-\alpha^2}{\beta+\gamma-2\alpha}, \frac{\gamma\alpha-\beta^2}{\gamma+\alpha-2\beta}, \frac{\alpha\beta-\gamma^2}{\alpha+\beta-2\gamma}$. [5]

Group-B
(25 Marks)

Answer any **five** questions

. $5 \times 5 = 25$

10. Prove that every permutation on a finite set is either a cycle or it can be expressed as a product of disjoint cycles. [5]

11. Show that the intersection of two subgroups of a group G is a subgroup of G. Is the result true in case of union? Justify your answer. [5]

12. Show that the order of a cyclic group is equal to the order of its generator. [5]

13. Prove that a non-trivial finite ring having no divisors of zero is a ring with unity. Can it be extended to an integral domain? Explain with an example. [5]

14. Define a subfield of a field. Prove that $\mathbb{Q}[\sqrt{2}]$ forms a subfield of $\mathbb{R}$. [5]

15. If $f : A \to B$ is a mapping and P, Q are nonempty subsets of A, then show that $f(P \cup Q) = f(P) \cup f(Q)$. [5]

16. Prove that every field is an integral domain. Is the converse true? Justify your answer with an example. [5]

Group-C
(25 Marks)

Answer any **five** questions

. $5 \times 5 = 25$

17. If $2s = a+b+c$, then show that $\begin{vmatrix} a^2 & (s-a)^2 & (s-a)^2 \\ (s-b)^2 & b^2 & (s-b)^2 \\ (s-c)^2 & (s-c)^2 & c^2 \end{vmatrix} = 2s^3(s-a)(s-b)(s-c)$. [5]

18. If $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$, then show that for every integer $n \geq 3$, $A^n = A^{n-2} + A^2 - I$. Hence evaluate A^{50}. [5]

19. Find the value of k, if the following system:

$$x - y + z = 2$$
$$2x + 3y - 4z = k$$
$$3x + 4y + 5z = k^2$$

has

(i) unique solution
(ii) infinitely many solutions
(iii) no solution. [5]

20. Use the Gram-Schmidt method of orthogonalization to construct an orthonormal basis for the subspace of $\mathbb{R}^4$ generated by $(1,1,0,1), (1,-2,0,0), (1,0,-1,2)$. [5]

21. Prove that row rank and column rank of a matrix are equal. [5]

22. Let $A = (a_{ij})_{n\times n}$ be a square matrix and A_{ij} be the co-factor of a_{ij} in $\det A$. Prove that $\det(A_{ij}) = (\det A)^{n-1}$. [5]

23. Let A be a 3×3 real matrix and $(1, 2, 1), (0, 1, 1), (1, 1, 1)$ be eigenvectors corresponding to the eigenvalues $2, 3, 1$ respectively. Find the matrix A. [5]

24. Let S denote the set of four vectors $(1, 2, -1), (-3, -6, 3), (2, 1, 3), (8, 7, 7)$. Find a linearly independent subset of S which spans the same space as S. [5]
