

RAIGANJ SURENDRANATH MAHAVIDYALAYA

DEPARTMENT OF MATHEMATICS

B.Sc (Hons) Semester-III Internal Examination, 2022.

Paper Code - MATH-H-DC-07

Time : 1:00 hour

Full Marks : 14

[CBCS]

Date : 21/01/2022

1. Answer all the following questions: $2 \times 3 = 6$

a. Show that the function $f(x, y) = \begin{cases} \frac{x^3 + y^3}{x - y}, & x \neq y \\ 0, & x = y \end{cases}$ is not continuous at $(0, 0)$.

b. If $v = \cos^{-1} \frac{x+y}{\sqrt{x} + \sqrt{y}}$, then prove that $x \frac{\partial v}{\partial x} + y \frac{\partial v}{\partial y} + \frac{1}{2} \cot v = 0$.

c. Show that the vector $y\hat{i} + x\hat{j}$ is both solenoidal and irrotational.

2. Answer all the following questions: $4 \times 2 = 8$

a. Show that the function $f(x, y) = \begin{cases} \frac{xy}{\sqrt{x^2 + y^2}}, & x^2 + y^2 \neq 0 \\ 0, & x = y = 0 \end{cases}$

is continuous, possesses partial derivatives but is not differentiable at the origin.

b. Verify Stokes theorem for the vector function

$\vec{F} = (x - 2y)\hat{i} + yz^2\hat{j} + y^2z\hat{k}$ where S denotes the upper half of the sphere $x^2 + y^2 + z^2 = 1$ and C is its boundary.

End

End